

From *The Mathematical Analysis of Logic* (1847) by George Boole

They who are acquainted with the present state of the theory of Symbolical Algebra, are aware, that the validity of the processes of analysis does not depend upon the interpretation of the symbols which are employed, but solely upon the laws of their combination. Every system of interpretation which does not affect the truth of the relations supposed, is equally admissible, and it is thus that the same process may, under one scheme of interpretation, represent the solution of a question on the properties of numbers, under another, that of a geometrical problem, and under a third, that of a problem of dynamics or optics. This principle is indeed of fundamental importance; and it may with safety be affirmed, that the recent advances of pure analysis have been much assisted by the influence which it has exerted in directing the current of investigation.

But the full recognition of the consequences of this important doctrine has been, in some measure, retarded by accidental circumstances. It has happened in every known form of analysis, that the elements to be determined have been conceived as measurable by comparison with some fixed standard. The predominant idea has been that of magnitude, or more strictly, of numerical ratio. The expression of magnitude, or of operations upon magnitude, has been the express object for which the symbols of Analysis have been invented, and for which their laws have been investigated. Thus the abstractions of the modern Analysis, not less than the ostensive diagrams of the ancient Geometry, have encouraged the notion, that Mathematics are essentially, as well as actually, the Science of Magnitude.

The consideration of that view which has already been stated, as embodying the true principle of the Algebra of Symbols, would, however, lead us to infer that this conclusion is by no means necessary. If every existing interpretation is shown to involve the idea of magnitude, it is only by induction that we can assert that no other interpretation is possible. And it may be doubted whether our experience is sufficient to render such an induction legitimate. The history of pure Analysis is, it may be said, too recent to permit us to set limits to the extent of its applications. Should we grant to the inference a high degree of probability, we might still, and with reason, maintain the sufficiency of the definition to which the principle already stated would lead us. We might justly assign it as the definitive character of a true Calculus, that it is a method resting upon the employment of Symbols, whose laws of combination are known and general, and whose results admit of a consistent interpretation. That to the existing forms of Analysis a quantitative interpretation is assigned, is the result of the circumstances by which those forms were determined, and is not to be construed into a universal condition of Analysis. It is upon the

foundation of this general principle, that I purpose to establish the Calculus of Logic, and that I claim for it a place among the acknowledged forms of Mathematical Analysis, regardless that in its object and in its instruments it must at present stand alone.

That which renders Logic possible, is the existence in our minds of general notions,—our ability to conceive of a class, and to designate its individual members by a common name. The theory of Logic is thus intimately connected with that of Language. A successful attempt to express logical propositions by symbols, the laws of whose combinations should be founded upon the laws of the mental processes which they represent, would, so far, be a step toward a philosophical language. But this is a view which we need not here follow into detail. Assuming the notion of a class, we are able, from any conceivable collection of objects, to separate by a mental act, those which belong to the given class, and to contemplate them apart from the rest. Such, or a similar act of election, we may conceive to be repeated. The group of individuals left under consideration may be still further limited, by mentally selecting those among them which belong to some other recognised class, as well as to the one before contemplated. And this process may be repeated with other elements of distinction, until we arrive at an individual possessing all the distinctive characters which we have taken into account, and a member, at the same time, of every class which we have enumerated. It is in fact a method similar to this which we employ whenever, in common language, we accumulate descriptive epithets for the sake of more precise definition.

Now the several mental operations which in the above case we have supposed to be performed, are subject to peculiar laws. It is possible to assign relations among them, whether as respects the repetition of a given operation or the succession of different ones, or some other particular, which are never violated. It is, for example, true that the result of two successive acts is unaffected by the order in which they are performed; and there are at least two other laws which will be pointed out in the proper place. These will perhaps to some appear so obvious as to be ranked among necessary truths, and so little important as to be undeserving of special notice. And probably they are noticed for the first time in this Essay. Yet it may with confidence be asserted, that if they were other than they are, the entire mechanism of reasoning, nay the very laws and constitution of the human intellect, would be vitally changed. A Logic might indeed exist, but it would no longer be the Logic we possess.

Such are the elementary laws upon the existence of which, and upon their capability of exact symbolical expression, the method of the following Essay is founded; and it is presumed that the

object which it seeks to attain will be thought to have been very fully accomplished. Every logical proposition, whether categorical or hypothetical, will be found to be capable of exact and rigorous expression, and not only will the laws of conversion and of syllogism be thence deducible, but the resolution of the most complex systems of propositions, the separation of any proposed element, and the expression of its value in terms of the remaining elements, with every subsidiary relation involved. Every process will represent deduction, every mathematical consequence will express a logical inference. The generality of the method will even permit us to express arbitrary operations of the intellect, and thus lead to the demonstration of general theorems in logic analogous, in no slight degree, to the general theorems of ordinary mathematics. No inconsiderable part of the pleasure which we derive from the application of analysis to the interpretation of external nature, arises from the conceptions which it enables us to form of the universality of the dominion of law. The general formulae to which we are conducted seem to give to that element a visible presence, and the multitude of particular cases to which they apply, demonstrate the extent of its sway. Even the symmetry of their analytical expression may in no fanciful sense be deemed indicative of its harmony and its consistency. Now I do not presume to say to what extent the same sources of pleasure are opened in the following Essay. The measure of that extent may be left to the estimate of those who shall think the subject worthy of their study. But I may venture to assert that such occasions of intellectual gratification are not here wanting. The laws we have to examine are the laws of one of the most important of our mental faculties. The mathematics we have to construct are the mathematics of the human intellect. Nor are the form and character of the method, apart from all regard to its interpretation, undeserving of notice. There is even a remarkable exemplification, in its general theorems, of that species of excellence which consists in freedom from exception. And this is observed where, in the corresponding cases of the received mathematics, such a character is by no means apparent. The few who think that there is that in analysis which renders it deserving of attention for its own sake, may find it worth while to study it under a form in which every equation can be solved and every solution interpreted. Nor will it lessen the interest of this study to reflect that every peculiarity which they will notice in the form of the Calculus represents a corresponding feature in the constitution of their own minds.

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The relation in which this Essay stands at once to Logic and to Mathematics, may further justify some notice of the question which has lately been revived, as to the relative value of the two studies in a liberal education. One of the chief objections which 'have been urged against the study

of Mathematics in general, is but another form of that which has been already considered with respect to the use of symbols in particular. And it need not here be further dwelt upon, than to notice, that if it avails anything, it applies with an equal force against the study of Logic. The canonical forms of the Aristotelian syllogism are really symbolical; only the symbols are less perfect of their kind than those of mathematics. If they are employed to test the validity of an argument, they as truly supersede the exercise of reason, as does a reference to a formula of analysis. Whether men do, in the present day, make this use of the Aristotelian canons, except as a special illustration of the rules of Logic, may be doubted; yet it cannot be questioned that when the authority of Aristotle was dominant in the schools of Europe, such applications were habitually made. And our argument only requires the admission, that the case is possible.

But the question before us has been argued upon higher grounds. Regarding Logic as a branch of Philosophy, and defining Philosophy as the "science of a real existence," and "the research of causes," and assigning as its main business the investigation of the "why," while Mathematics display only the "that," Sir W. Hamilton has contended, not simply, that the superiority rests with the study of Logic, but that the study of Mathematics is at once dangerous and useless. The pursuits of the mathematician "have not only not trained him to that acute scent, to that delicate, almost instinctive, tact which, in the twilight of probability, the search and discrimination of its finer facts demand; they have gone to cloud his vision, to indurate his touch, to all but the blazing light, the iron chain of demonstration, and left him out of the narrow confines of his science, to a passive credulity in any premises, or to an absolute incredulity in all." In support of these and of other charges, both argument and copious authority are adduced. I shall not attempt a complete discussion of the topics which are suggested by these remarks. My object is not controversy, and the observations which follow are offered not in the spirit of antagonism, but in the hope of contributing to the formation of just views upon an important subject. Of Sir W. Hamilton it is impossible to speak otherwise than with that respect which is due to genius and learning.

Philosophy is then described as the science of a real existence and the research of causes. And that no doubt may rest upon the meaning of the word cause, it is further said, that philosophy "mainly investigates the why." These definitions are common among the ancient writers. Thus Seneca, one of Sir W. Hamilton's authorities, Epistle LXXXVIII., "The philosopher seeks and knows the causes of natural things, of which the mathematician searches out and computes the numbers and the measures." It may be remarked, in passing, that in whatever degree the belief has prevailed, that the business of philosophy is immediately with causes; in the same degree has every science

whose object is the investigation of laws, been lightly esteemed. Thus the Epistle to which we have referred, bestows, by contrast with Philosophy, a separate condemnation on Music and Grammar, on Mathematics and Astronomy, although it is that of Mathematics only that Sir W. Hamilton has quoted.

Now we might take our stand upon the conviction of many thoughtful and reflective minds, that in the extent of the meaning above stated, Philosophy is impossible. The business of true Science, they conclude, is with laws and phenomena. The nature of Being, the mode of the operation of Cause, the why, they hold to be beyond the reach of our intelligence. But we do not require the vantage-ground of this position; nor is it doubted that whether the aim of Philosophy is attainable or not, the desire which impels us to the attempt is an instinct of our higher nature. Let it be granted that the problem which has baffled the efforts of ages, is not a hopeless one; that the "science of a real existence," and "the research of causes," "that kernel" for which "Philosophy is still militant," do not transcend the limits of the human intellect. I am then compelled to assert, that according to this view of the nature of Philosophy, Logic forms no part of it. On the principle of a true classification, we ought no longer to associate Logic and Metaphysics, but Logic and Mathematics.

Should any one after what has been said, entertain a doubt upon this point, I must refer him to the evidence which will be afforded in the following Essay. He will there see Logic resting like Geometry upon axiomatic truths, and its theorems constructed upon that general doctrine of symbols, which constitutes the foundation of the recognised Analysis. In the Logic of Aristotle he will be led to view a collection of the formulae of the science, expressed by another, but, (it is thought) less perfect scheme of symbols. I feel bound to contend for the absolute exactness of this parallel. It is no escape from the conclusion to which it points to assert, that Logic not only constructs a science, but also inquires into the origin and the nature of its own principles,—a distinction which is denied to Mathematics. "It is wholly beyond the domain of mathematicians," it is said, "to inquire into the origin and nature of their principles."— Review, page 415. But upon what ground can such a distinction be maintained? What definition of the term Science will be found sufficiently arbitrary to allow such differences?

The application of this conclusion to the question before us is clear and decisive. The mental discipline which is afforded by the study of Logic, as an exact science, is, in species, the same as that afforded by the study of Analysis.

Is it then contended that either Logic or Mathematics can supply a perfect discipline to the Intellect? The most careful and unprejudiced examination of this question leads me to doubt whether such a position can be maintained. The exclusive claims of either must, I believe, be abandoned, nor can any others, partaking of a like exclusive character, be admitted in their room. It is an important observation, which has more than once been made, that it is one thing to arrive at correct premises, and another thing to deduce logical conclusions, and that the business of life depends more upon the former than upon the latter. The study of the exact sciences may teach us the one, and it may give us some general preparation of knowledge and of practice for the attainment of the other, but it is to the union of thought with action, in the field of Practical Logic, the arena of Human Life, that we are to look for its fuller and more perfect accomplishment.

I desire here to express my conviction, that with the advance of our knowledge of all true science, an ever-increasing harmony will be found to prevail among its separate branches. The view which leads to the rejection of one, ought, if consistent, to lead to the rejection of others. And indeed many of the authorities which have been quoted against the study of Mathematics, are even more explicit in their condemnation of Logic. "Natural science," says the Chian Aristo, "is above us, Logical science does not concern us." When such conclusions are founded (as they often are) upon a deep conviction of the preeminent value and importance of the study of Morals, we admit the premises, but must demur to the inference. For it has been well said by an ancient writer, that it is the "characteristic of the liberal sciences, not that they conduct us to Virtue, but that they prepare us for Virtue;" and Melancthon's sentiment, "abeunt studia in mores," has passed into a proverb. Moreover, there is a common ground upon which all sincere votaries of truth may meet, exchanging with each other the language of Flamsted's appeal to Newton, "The works of the Eternal Providence will be better understood through your labors and mine."